CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

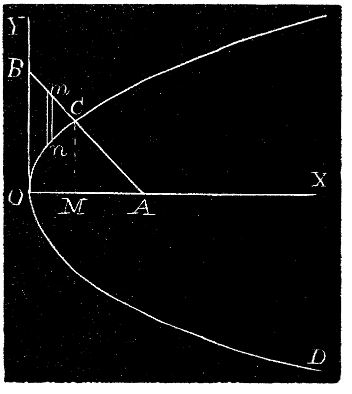
SOLUTION TO PROBLEMS.

I. Find the moment of inertia about the origin, of the area included within the parabola $y^2=4ax$, the line $x+y=3a$, and the axis of x.

[Selected from *Osborne's Differential and Integral Calculus.*]

I. **Solution by J. M. BANDY, Professor of Mathematics, Elon College, North Carolina.**

The equation of the parabola is $y^2=4ax$, and that of the given line $y=-x+3a$. Combining these equations, we get $x=OM=a$, and $y=CM=2\sqrt{ax}$. $\therefore$ for figure OMC, moment $=\int_0^a\int_0^{2\sqrt{a}}(x^2+y^2)dxdy$.



Again , for fig. MCA, x varies between a and $3a$, and y varies between 0 and $3a-x$.

$$\therefore \text{Moment}=\int_0^{3a}\int_0^{3a-x}(x^2+y^2)dxdy,$$

$\therefore$ for fig. OCA, moment $=\int_0^a\int_0^{2\sqrt{(ax)}}(x^2+y^2)dxdy+\int_a^{3a}\int_0^{3a-x}(x^2+y^2)dxdy.$

$$=\int_0^{2a}\int_{\frac{y^2}{4a}}^{3a-y}(x^2+y^2)dydx=\frac{314a^4}{35}.$$

II. **Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi, University, Mississippi.**

We wish to find the moment of inertia of AC_nO about OY.

The co-ordinates of C, the point of intersection of $y^2=4ax$ and $x+y=4a$, are $[2a(3-\sqrt{5}),\ 2a(\sqrt{5}-1)]$. Also $OB=OA=4a$. If I, I_1, and I_2 are the moments of inertia about OY of O_nCA, BOA, and BO_nC, respectively, $I=I_1-I_2$. Dividing BO_nC into elementary strips, such as mn, parallel to OY,

$$I_2=\int_0^{2a(3-\sqrt{5})}(-x+4a-2\sqrt{ax})x^2dx,$$

$$=\tfrac{32}{21}(213-95\sqrt{5})a^4.$$

$$I_1=\tfrac{1}{6}.\frac{OA^2}{2}.OA^2=\frac{64}{3}a^4.$$

$$\therefore\ I=\tfrac{64}{3}a^4-\tfrac{32}{21}(213-95\sqrt{5})a^4.$$

$$=\tfrac{32}{21}(95\sqrt{5}-199)a^4.$$

[The equation of the line in *Osborne* is $x+y=3a$, and not $x+y=4a$, as first printed in the MONTHLY. Prof. Bandy solves it for the first case and Prof. Hume for the second. Also solved by Professors *H. C. Whitaker*, *P. H. Philbrick*, *J. F. W. Scheffer* and by Cadet *A. J. Vaughan*, of the Virginia Military Institute, Lexington, Virginia.]